Lecture 5

Variants of Turing Machines

Time-Constructible Functions

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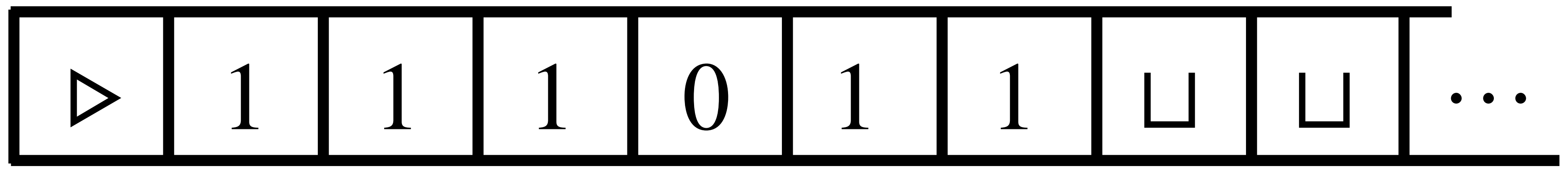
Input Tape



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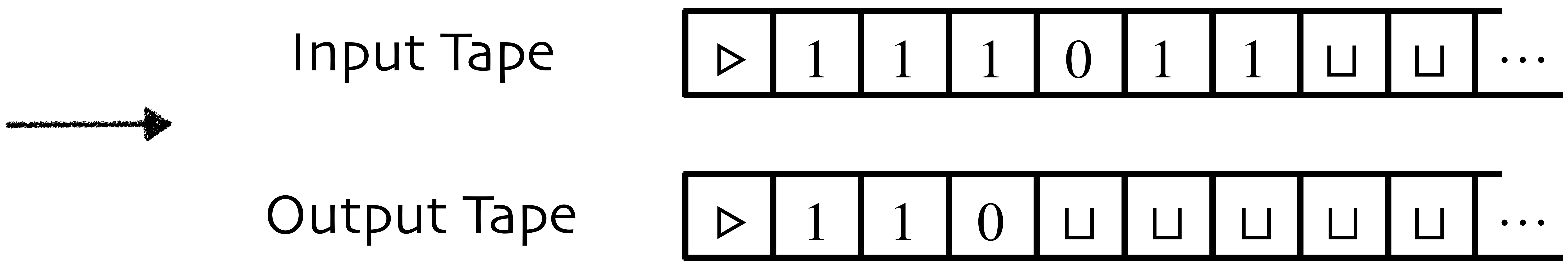

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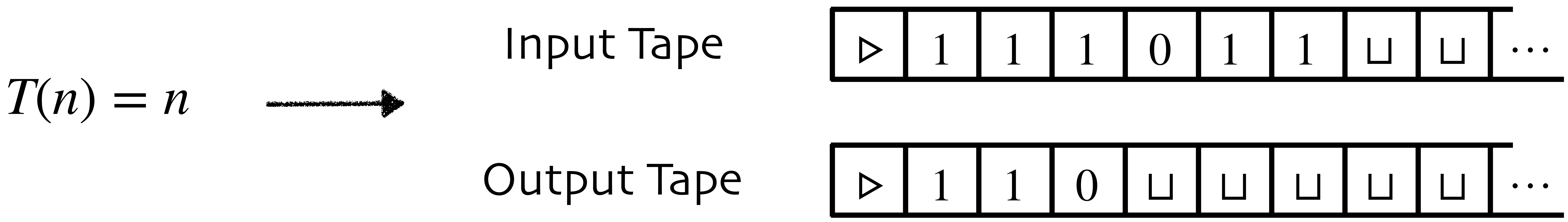

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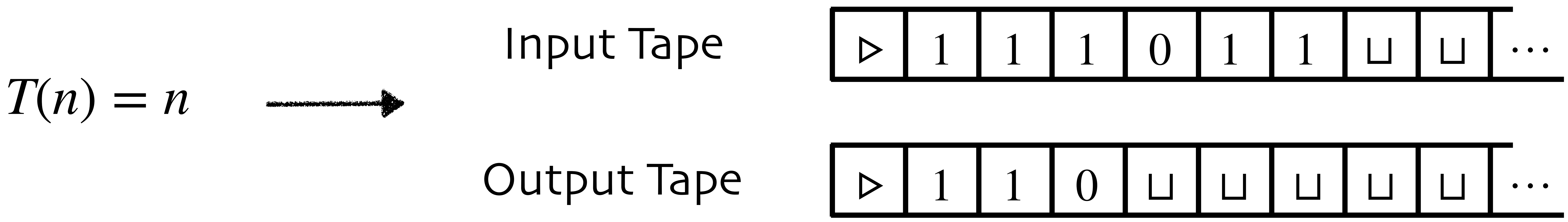

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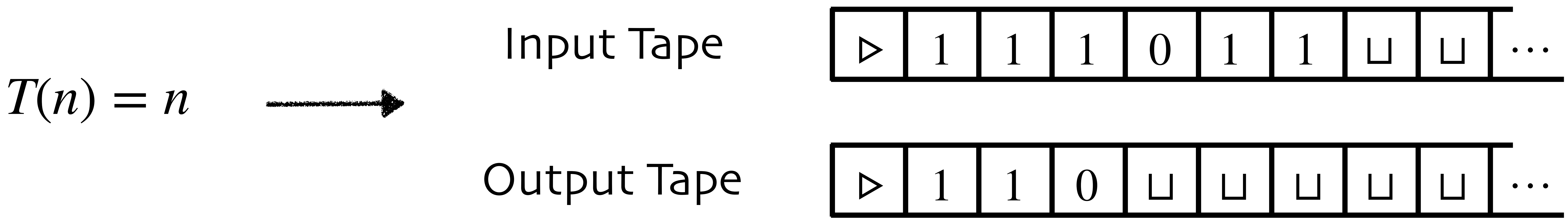

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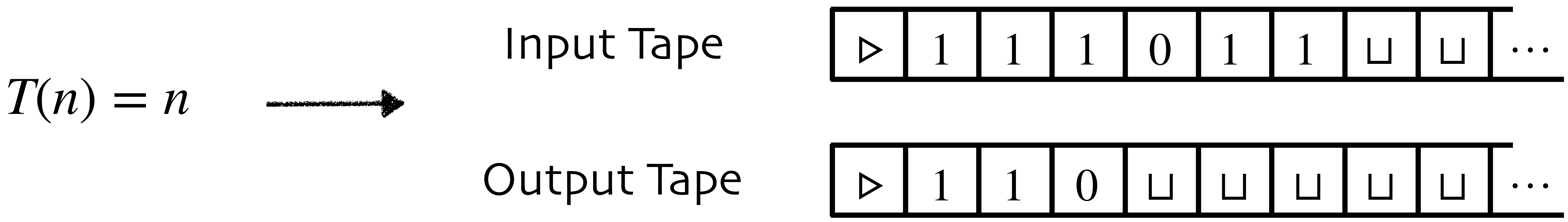

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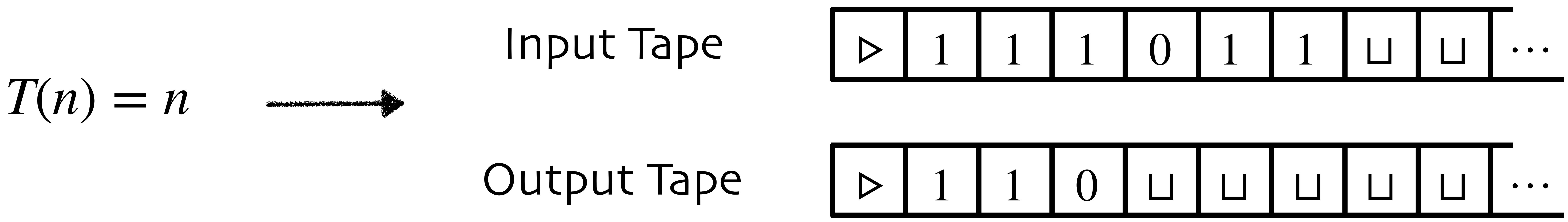

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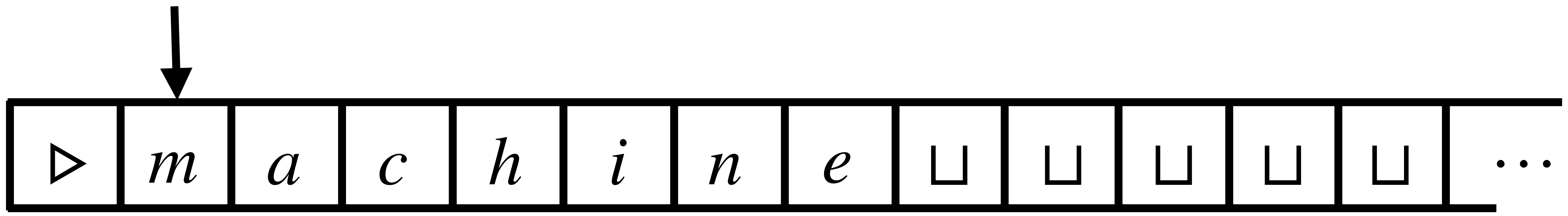

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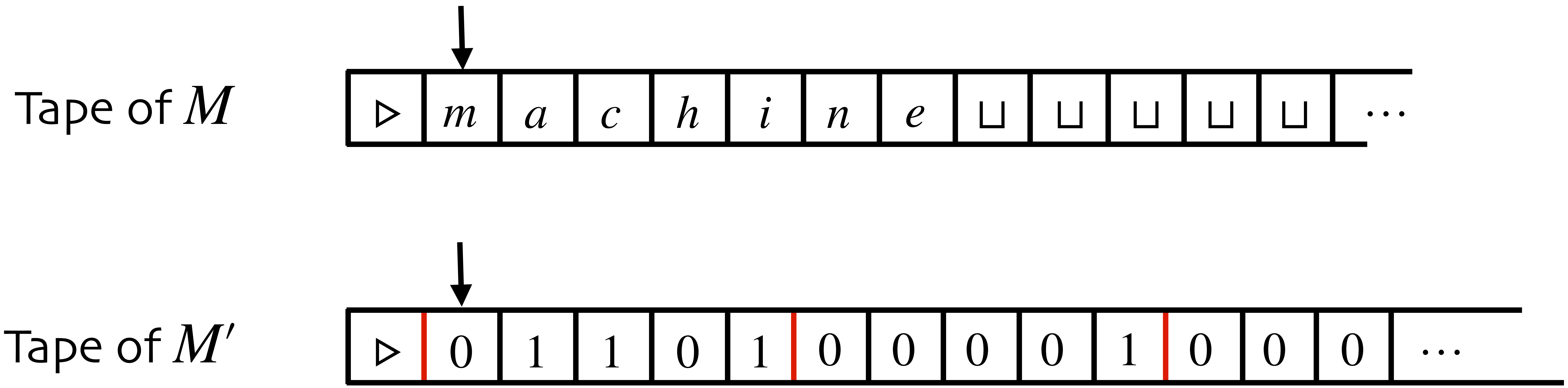

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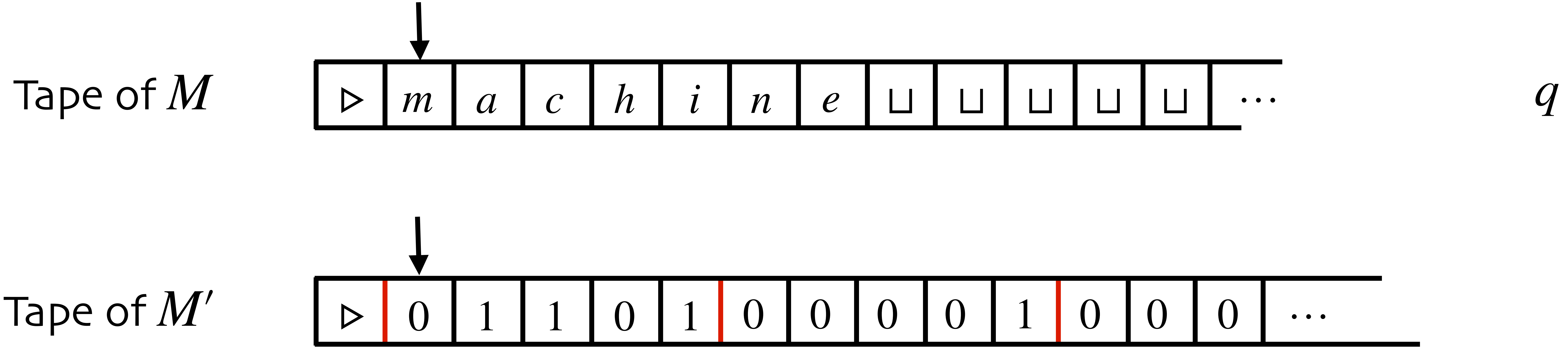

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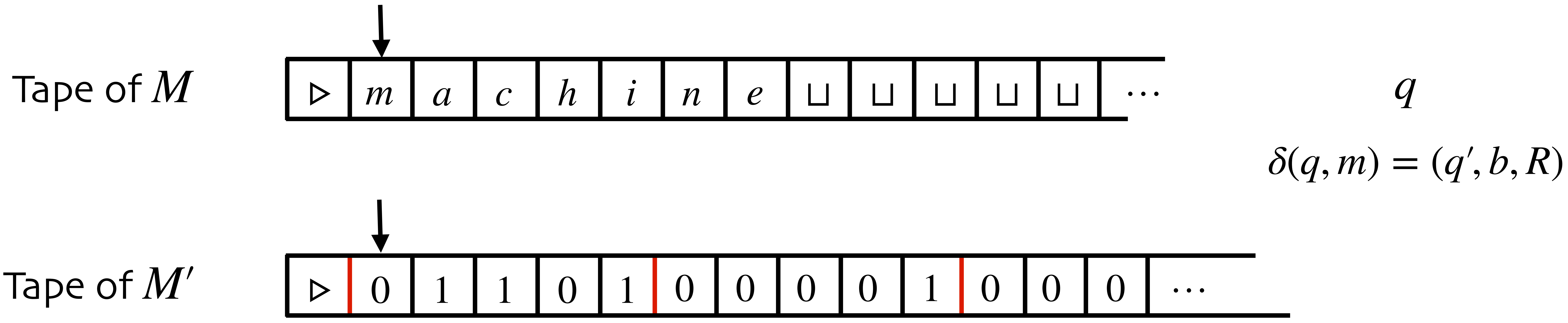

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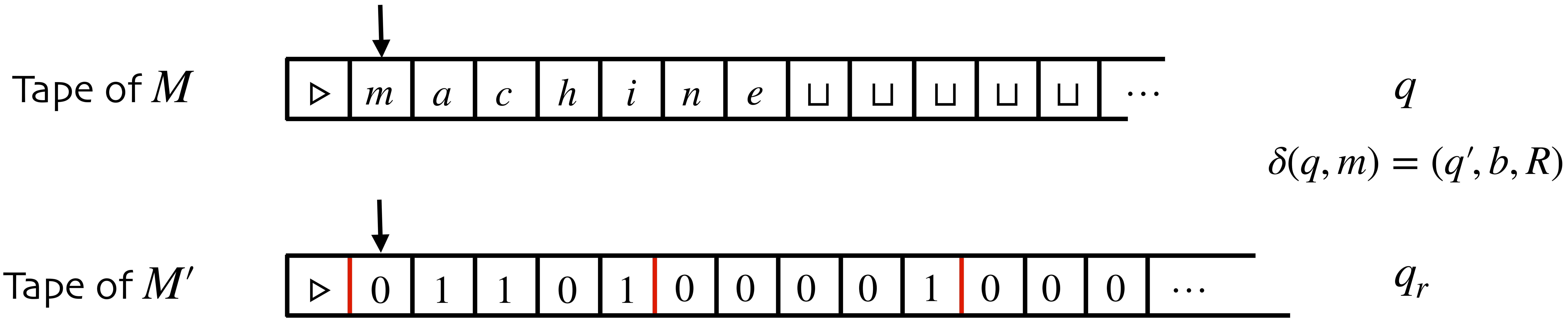

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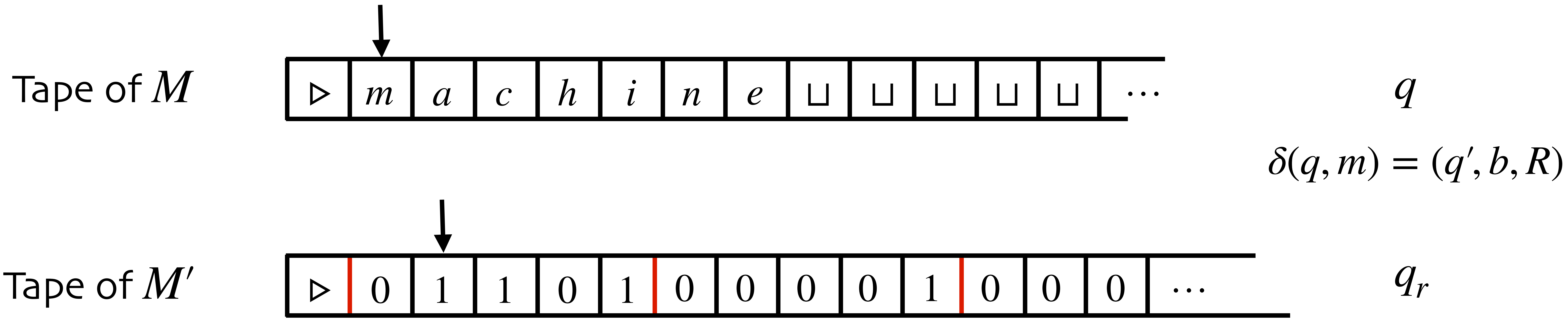

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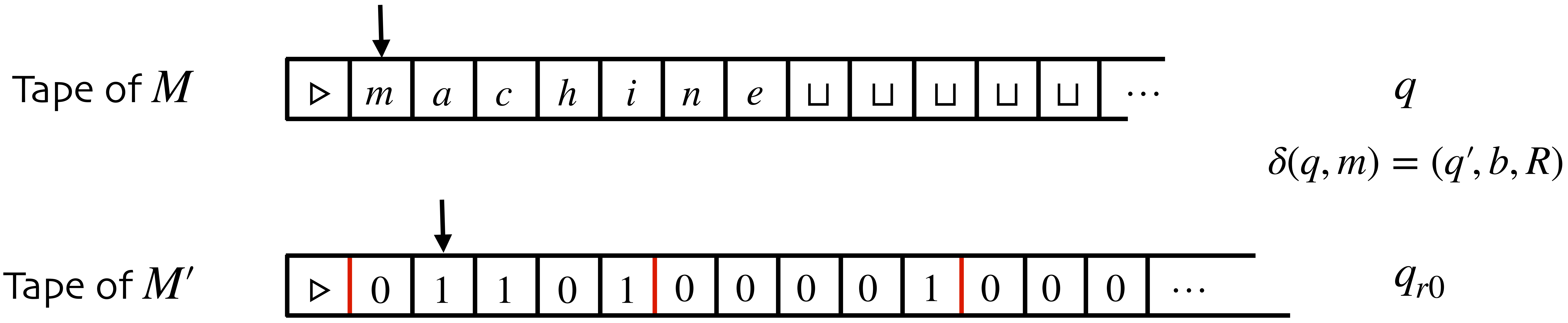

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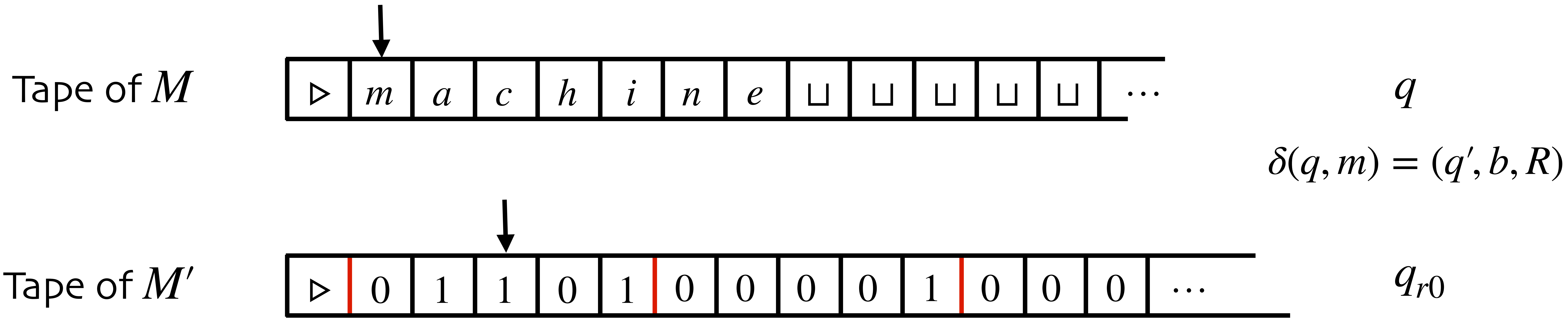

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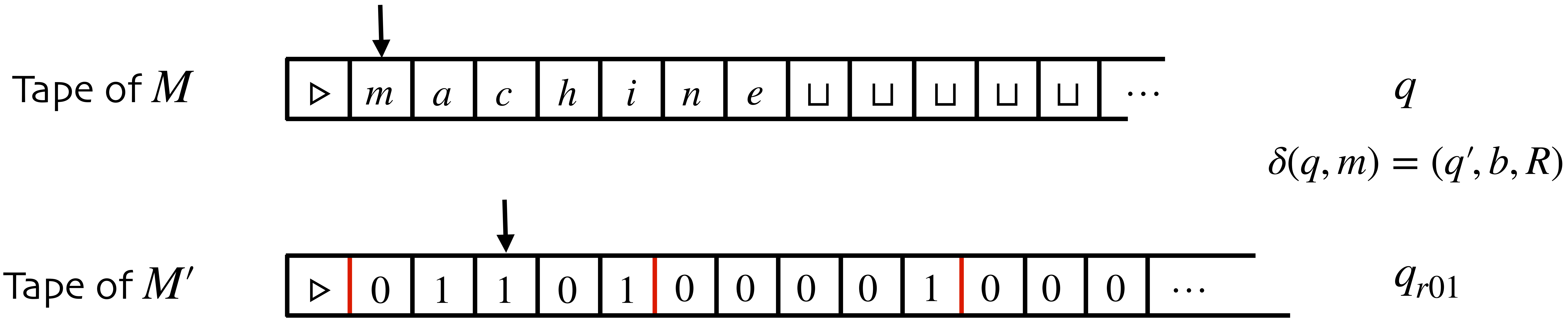

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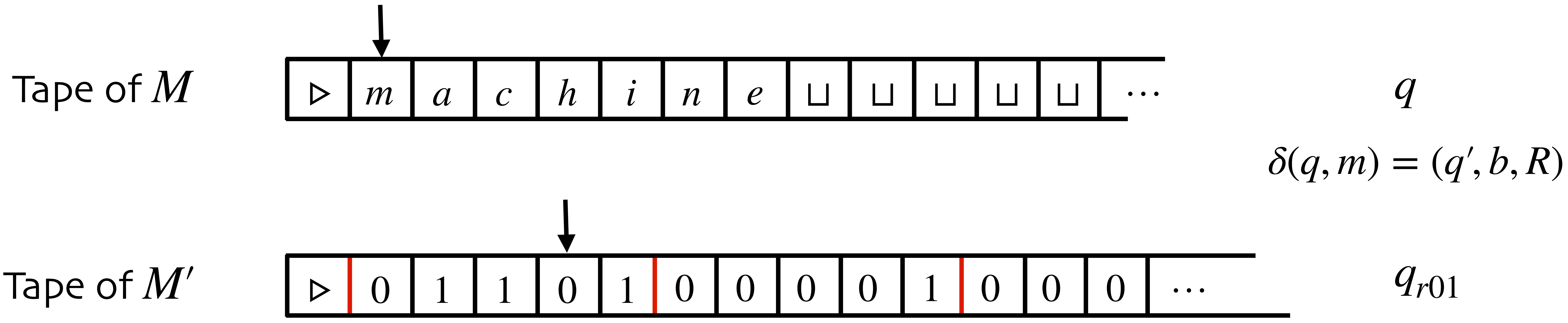

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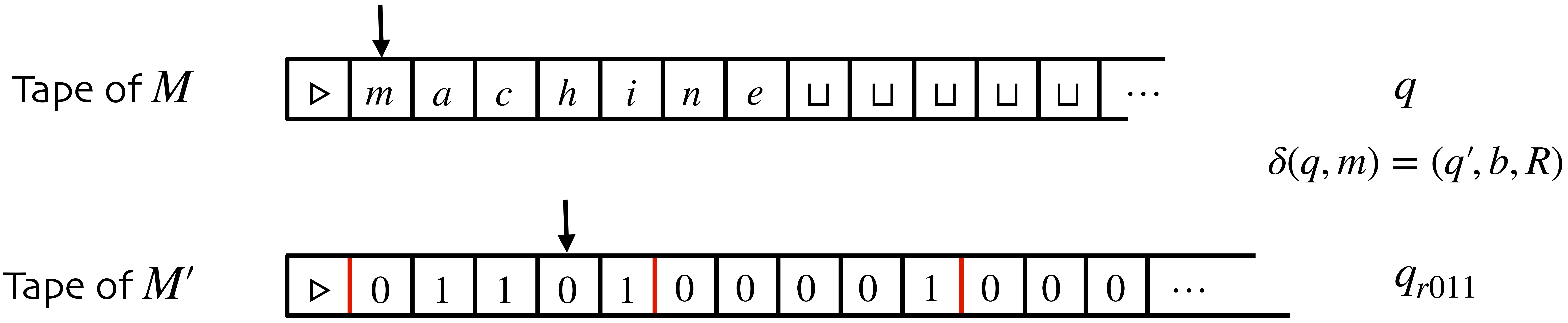

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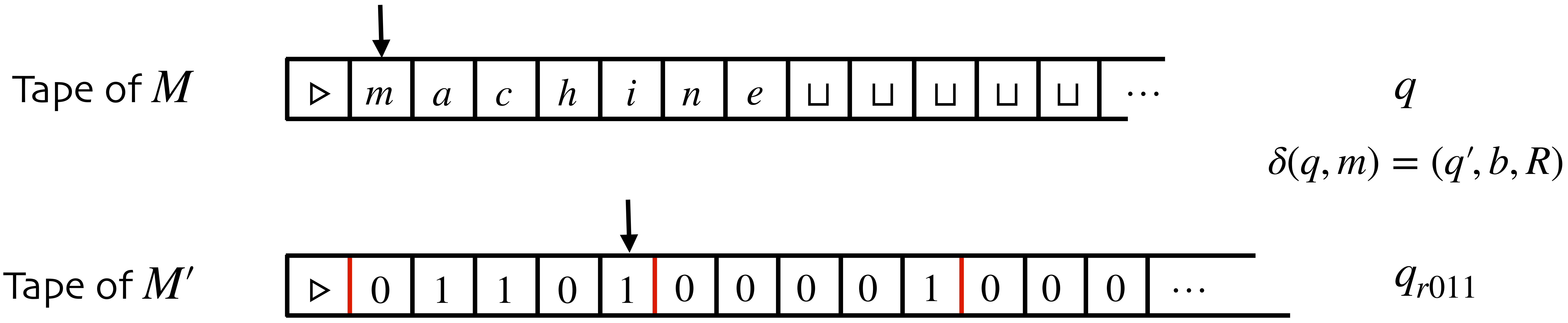

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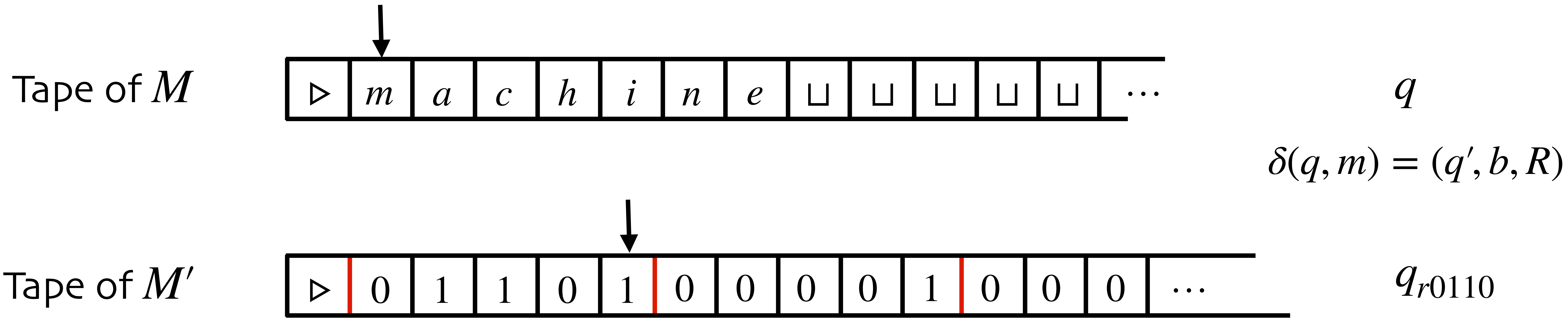

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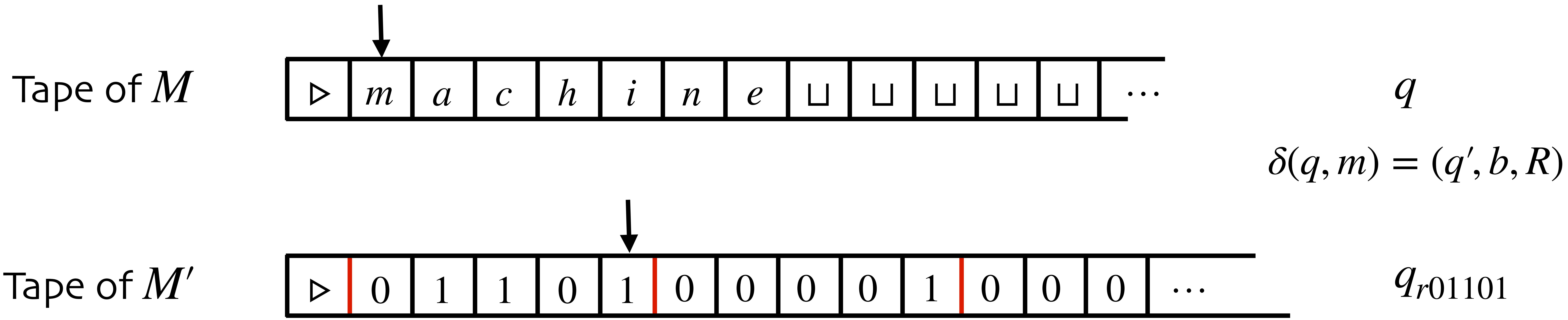

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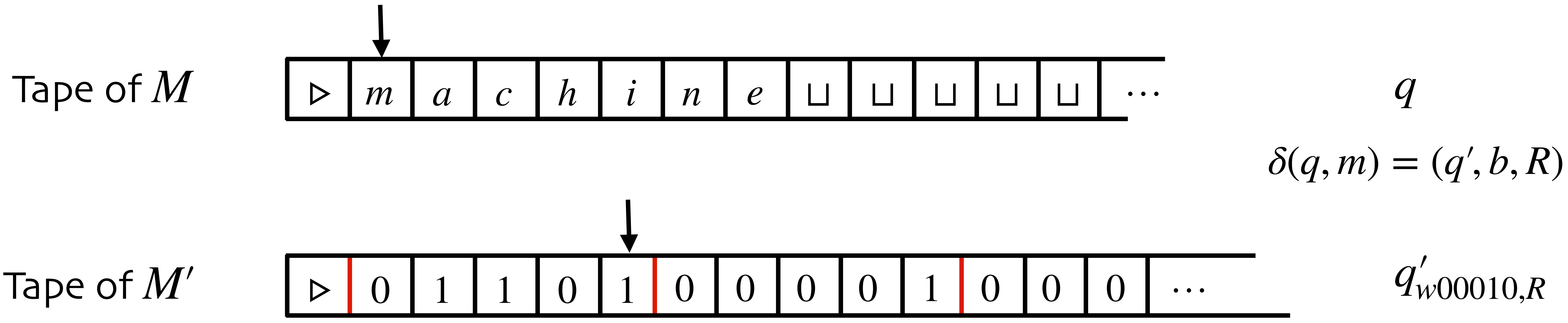

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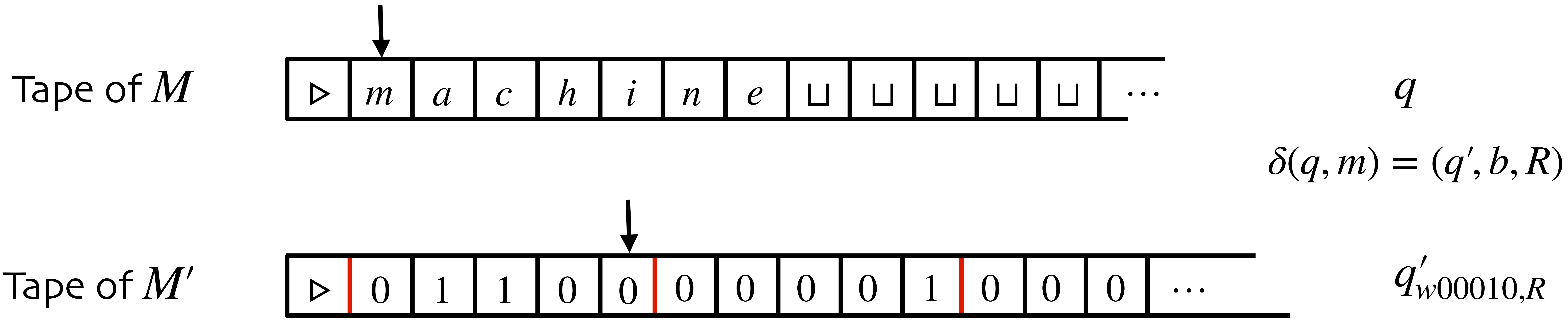

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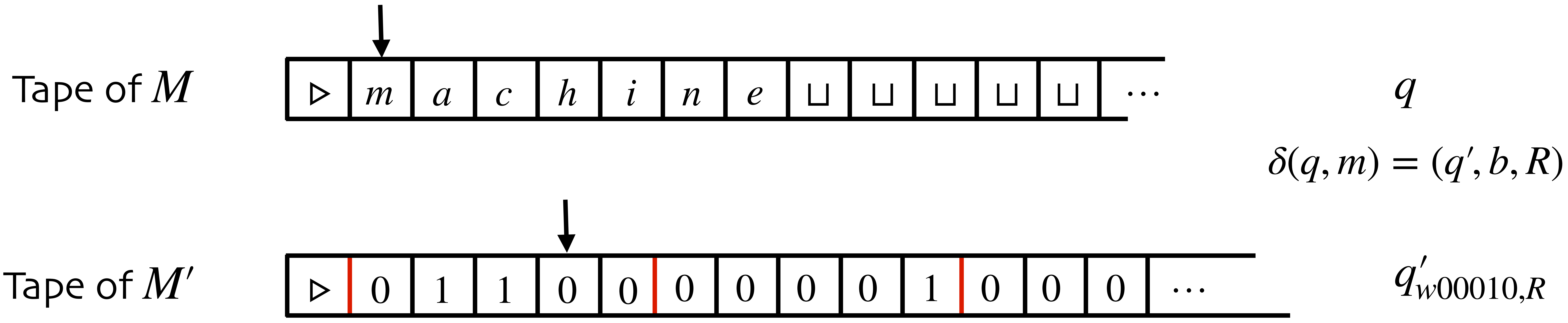

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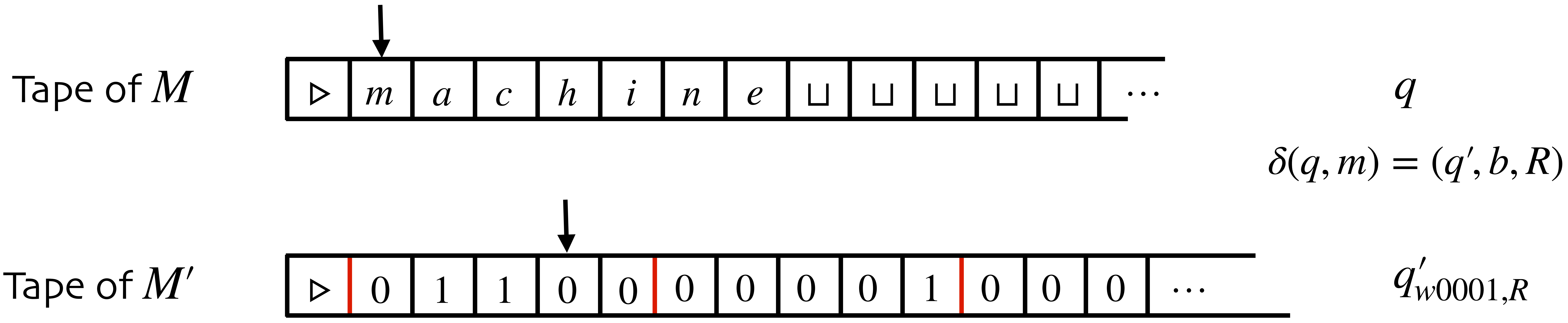

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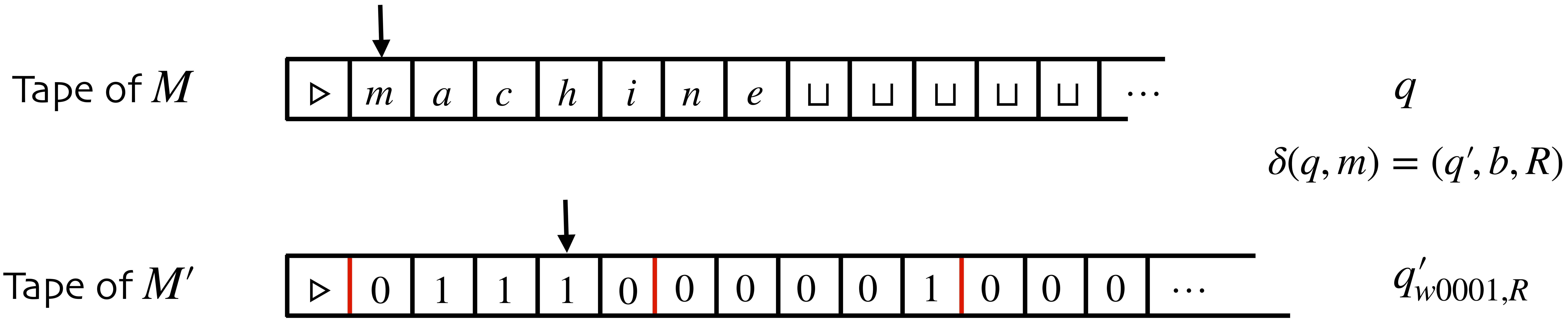

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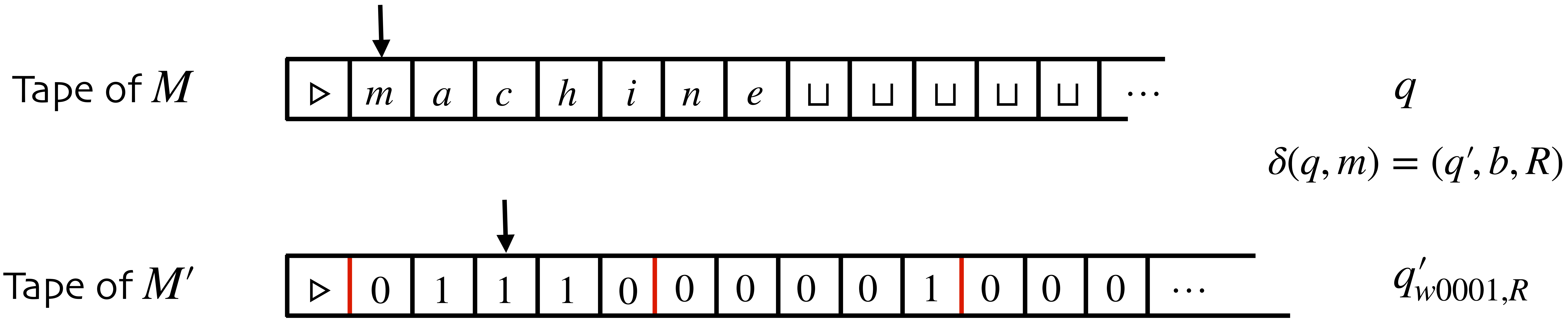

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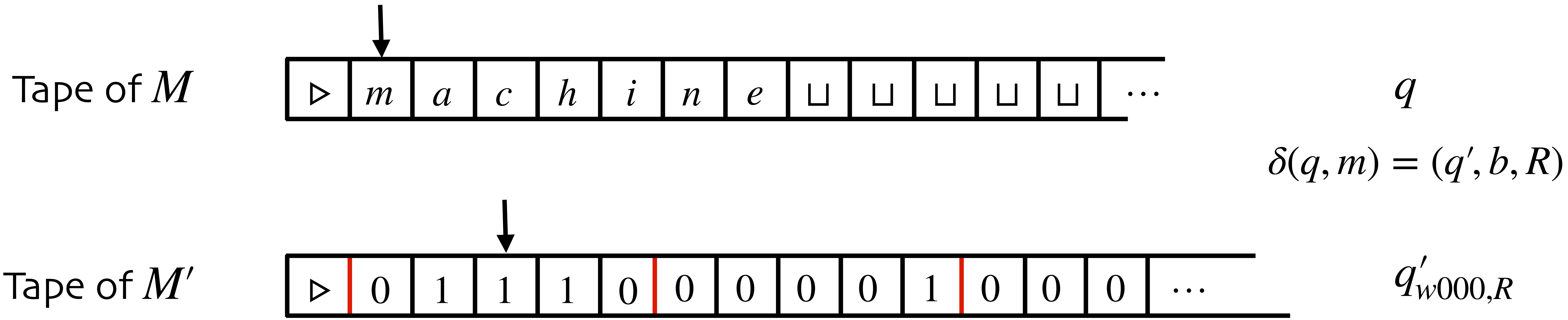

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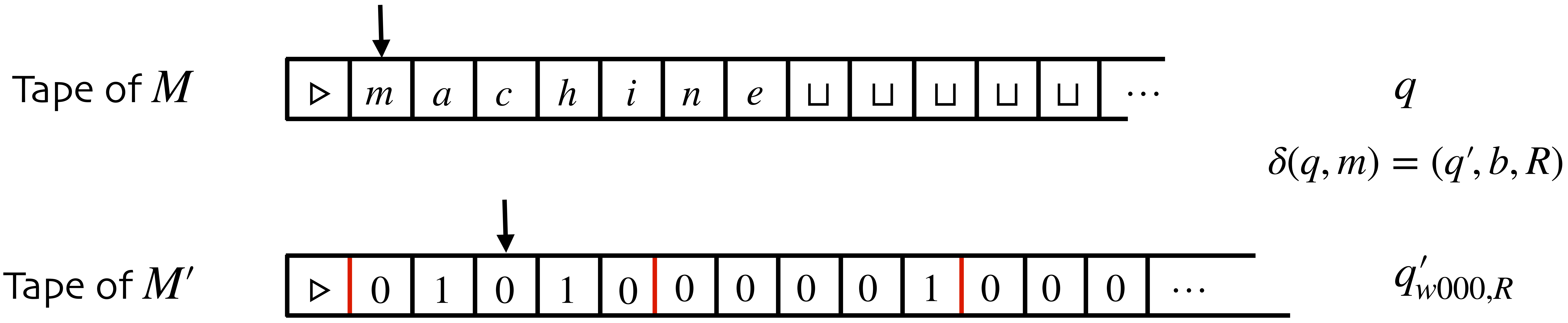

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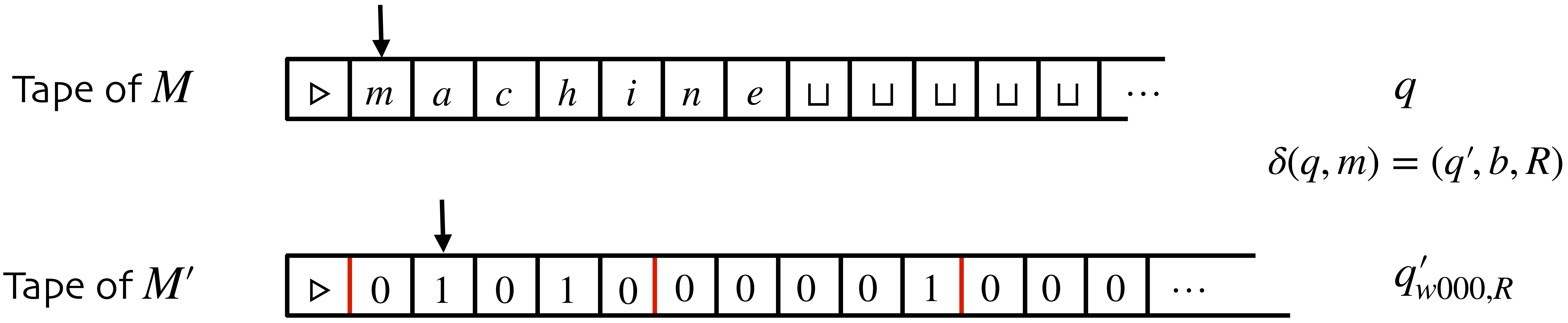

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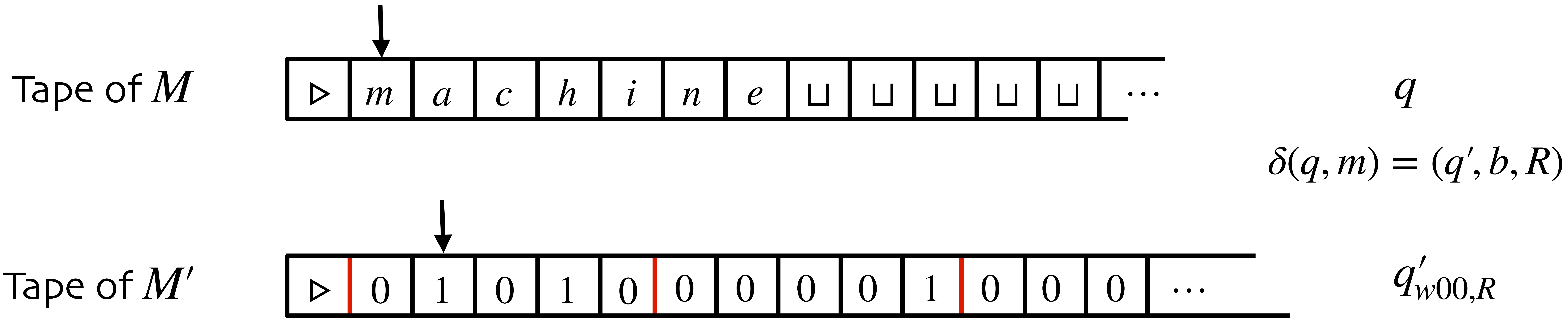

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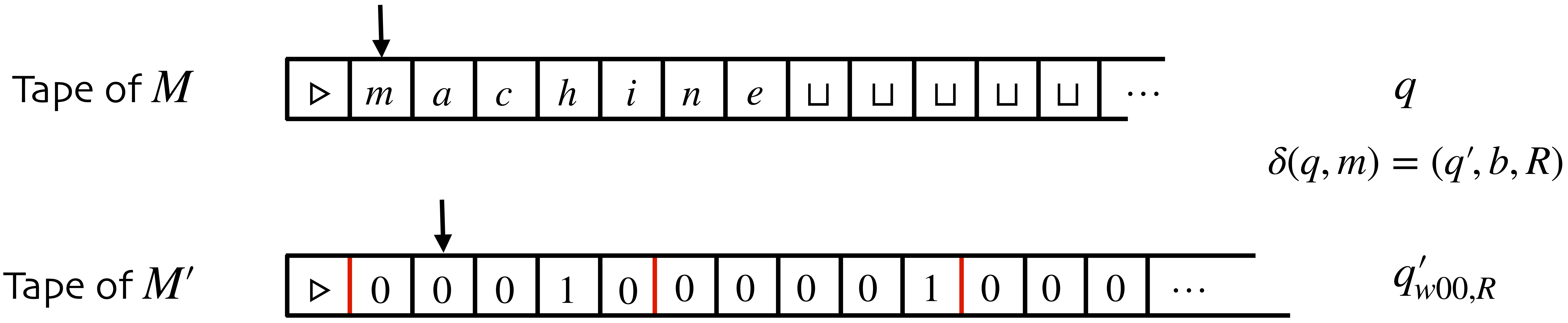

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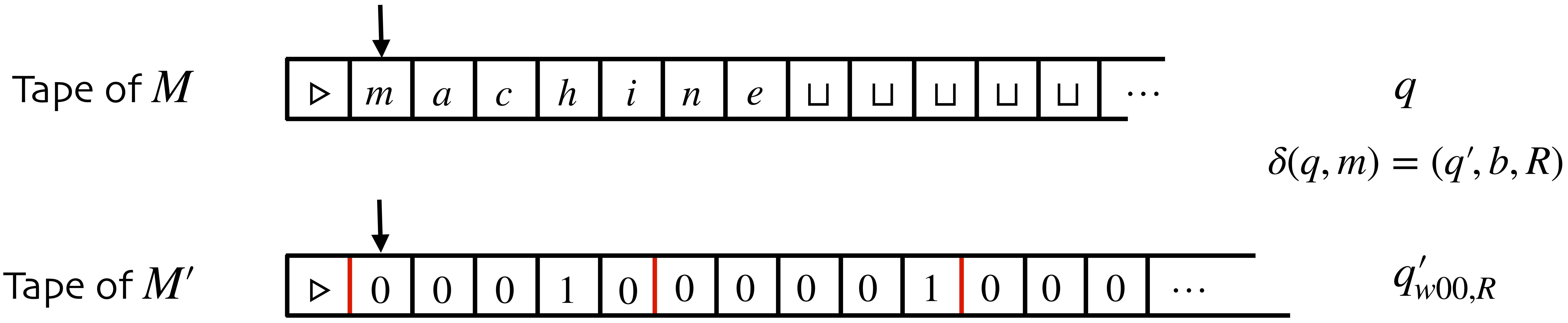

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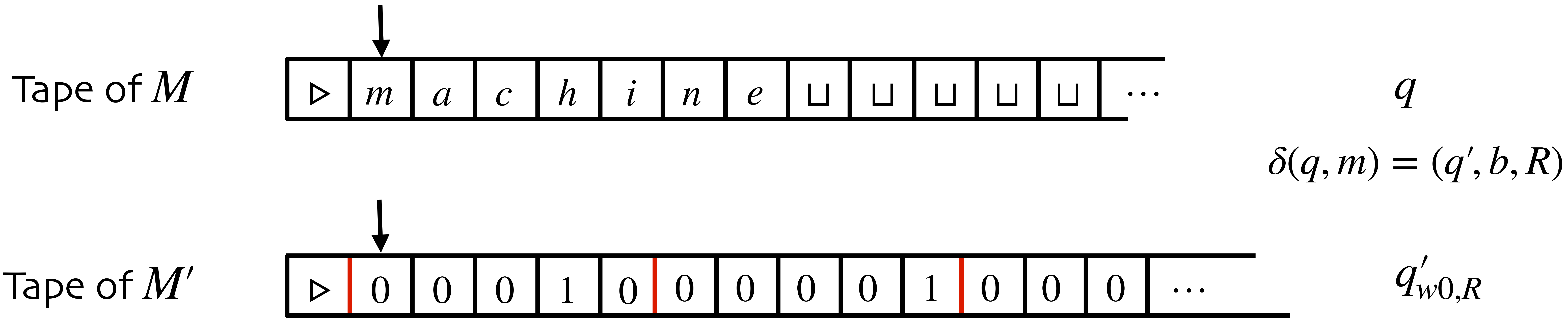

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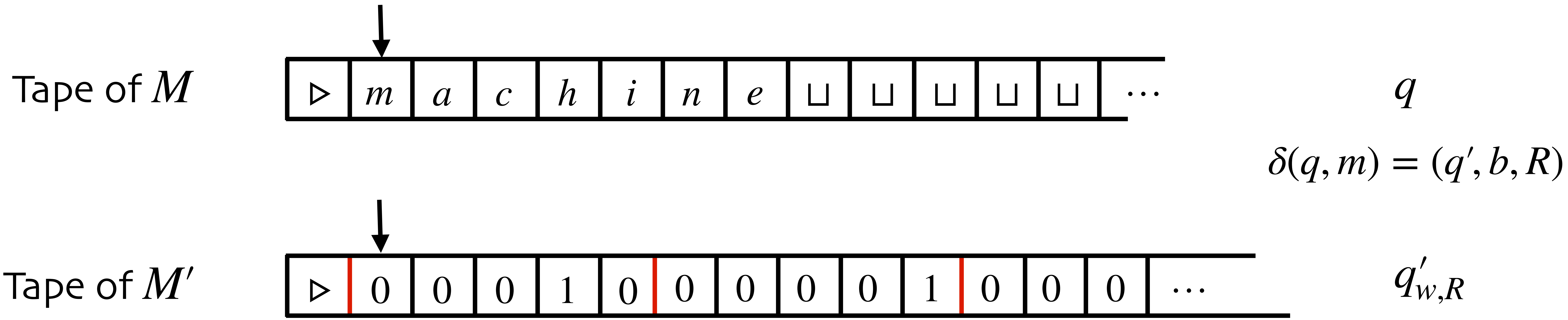

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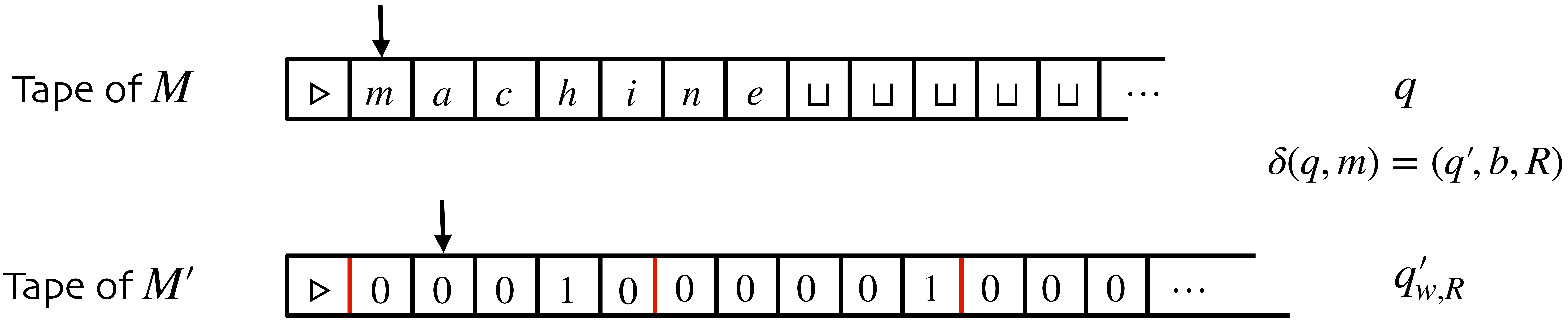

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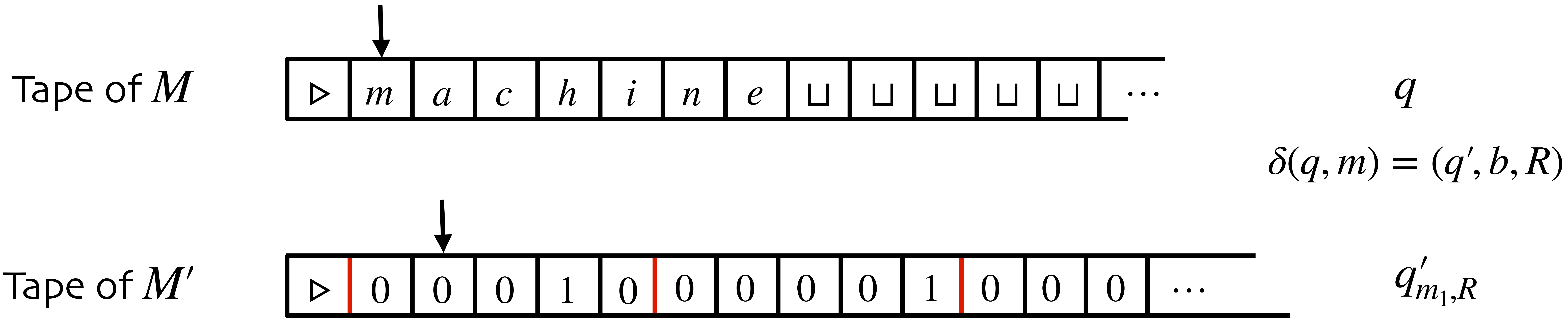

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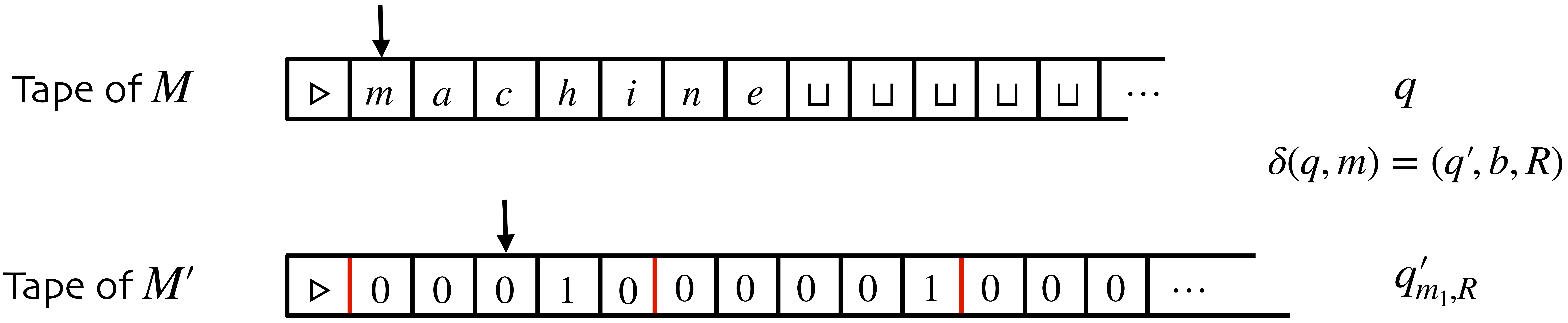

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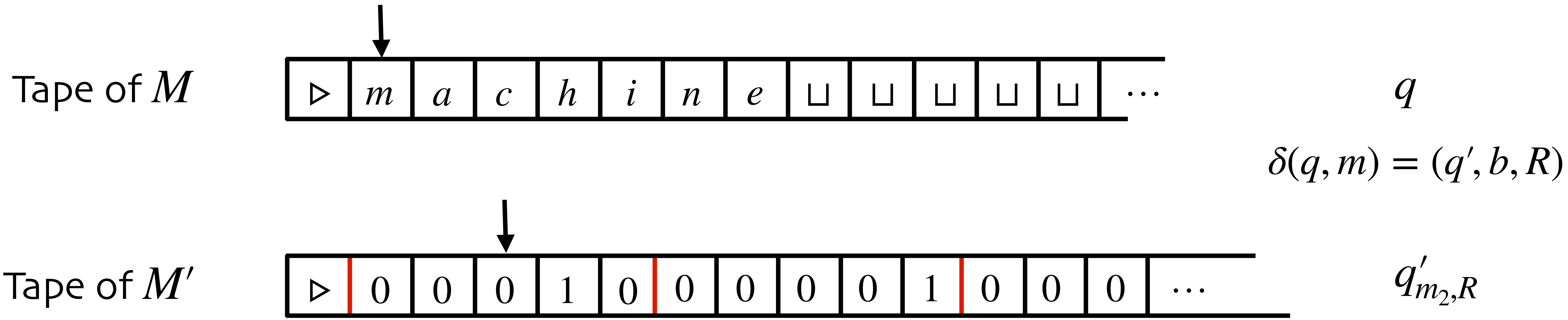

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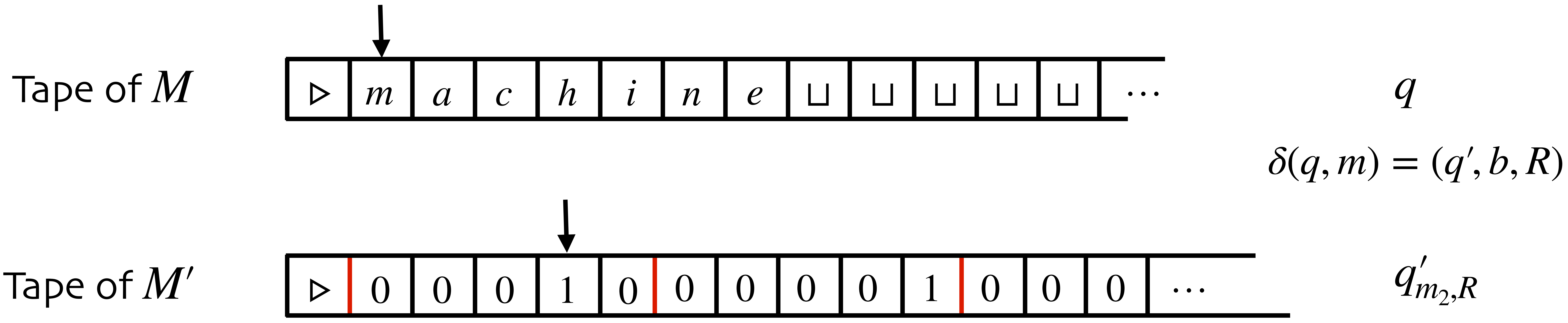

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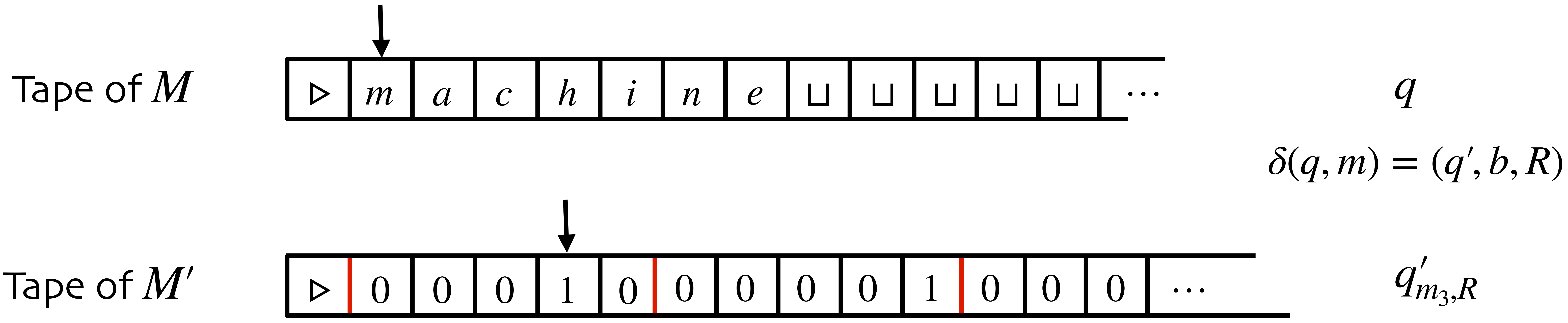

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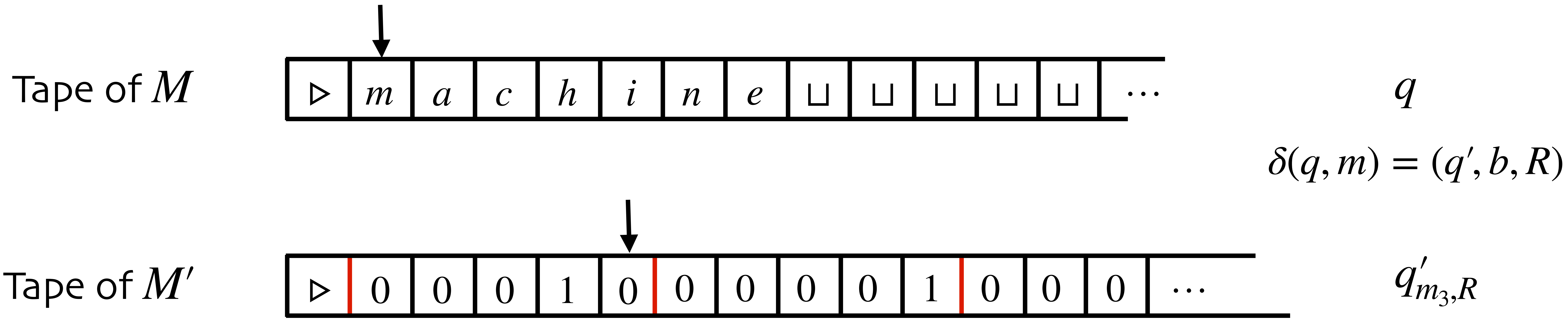

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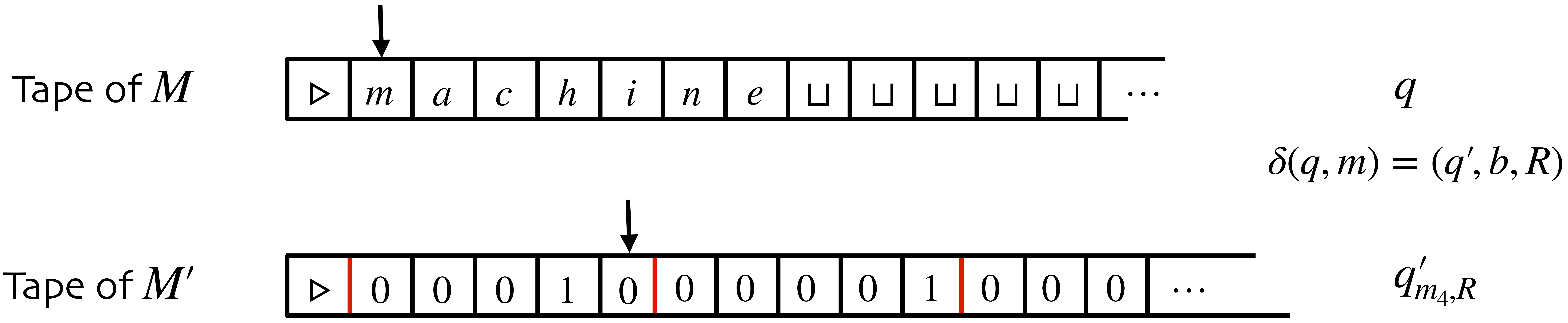

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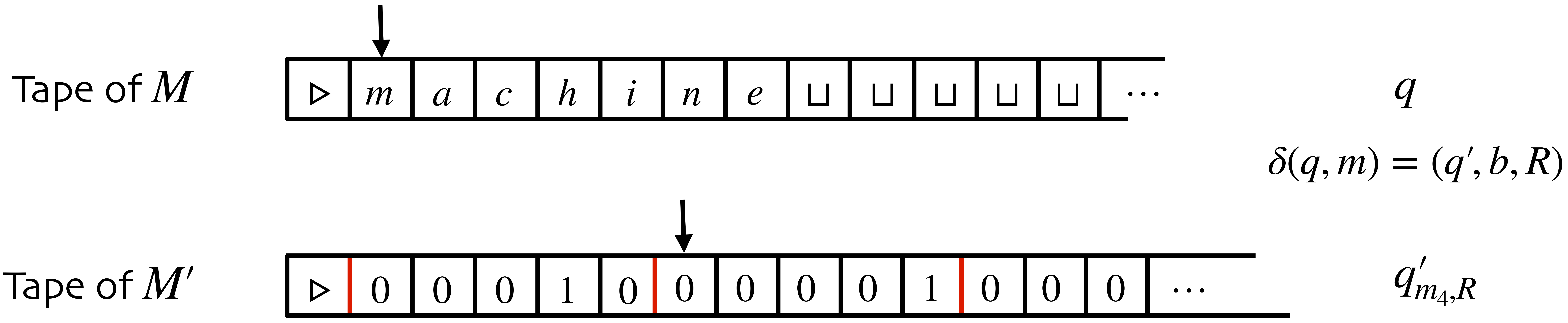

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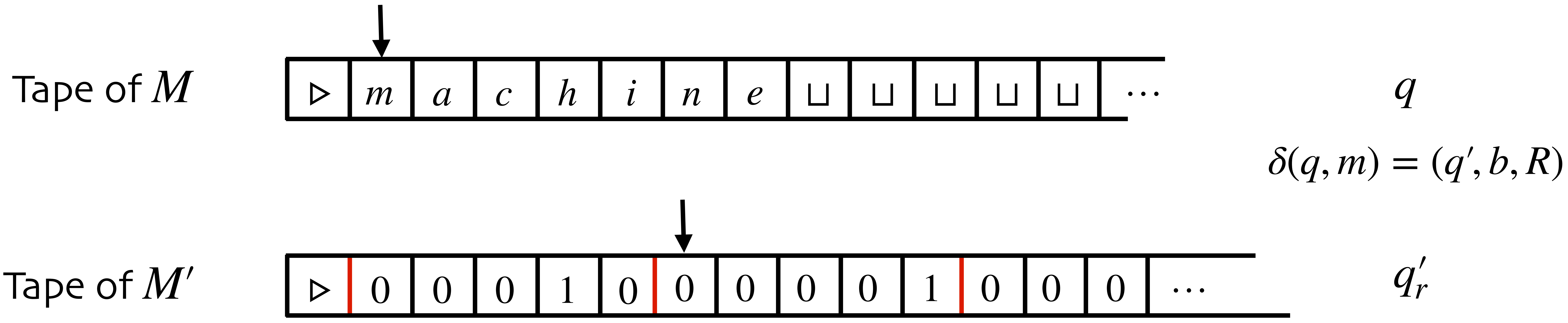

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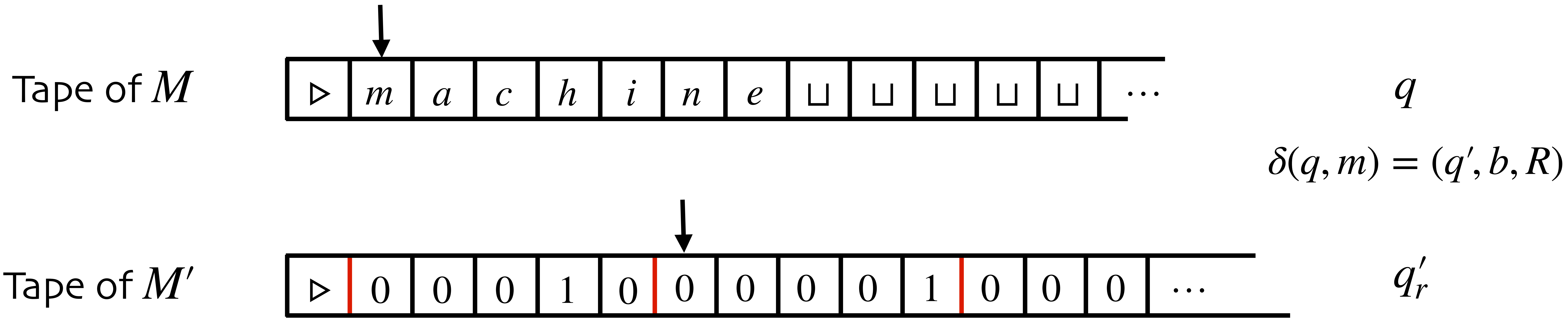

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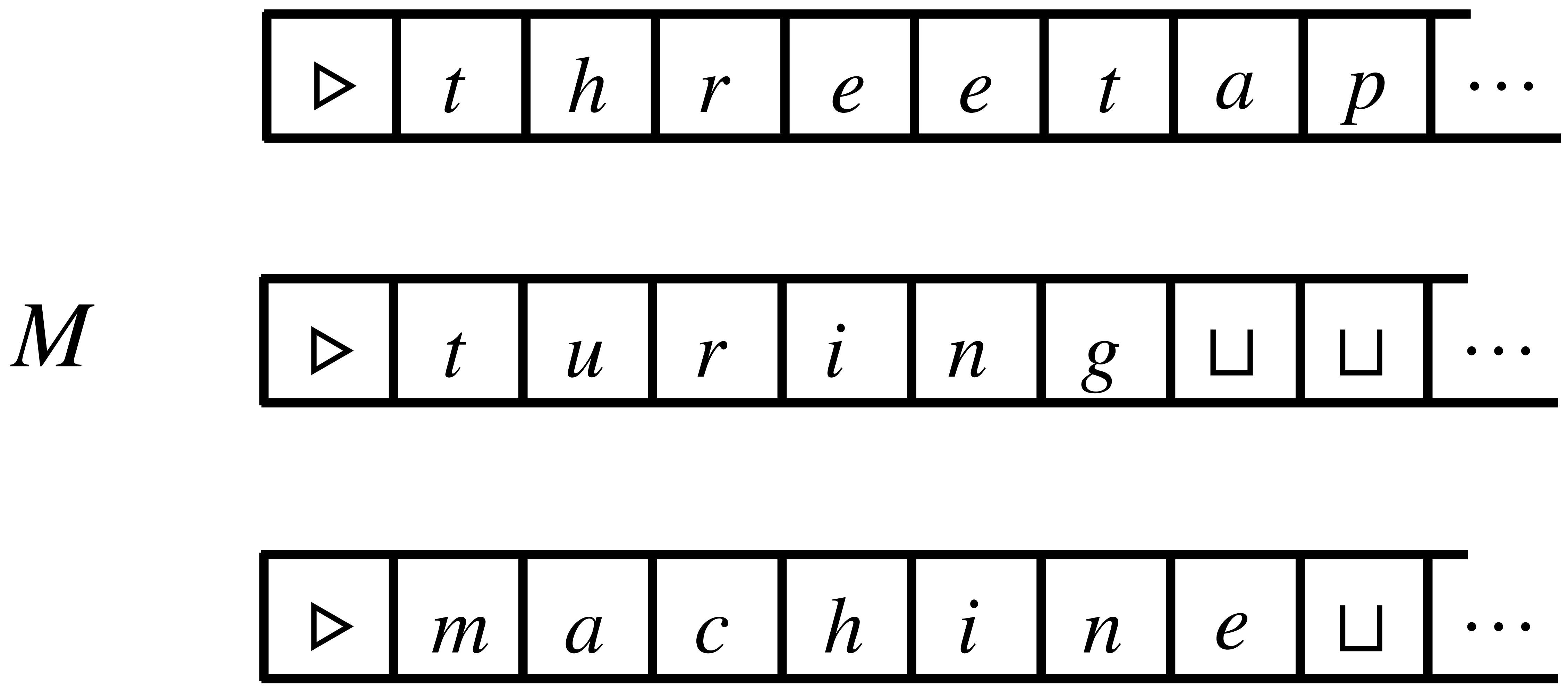

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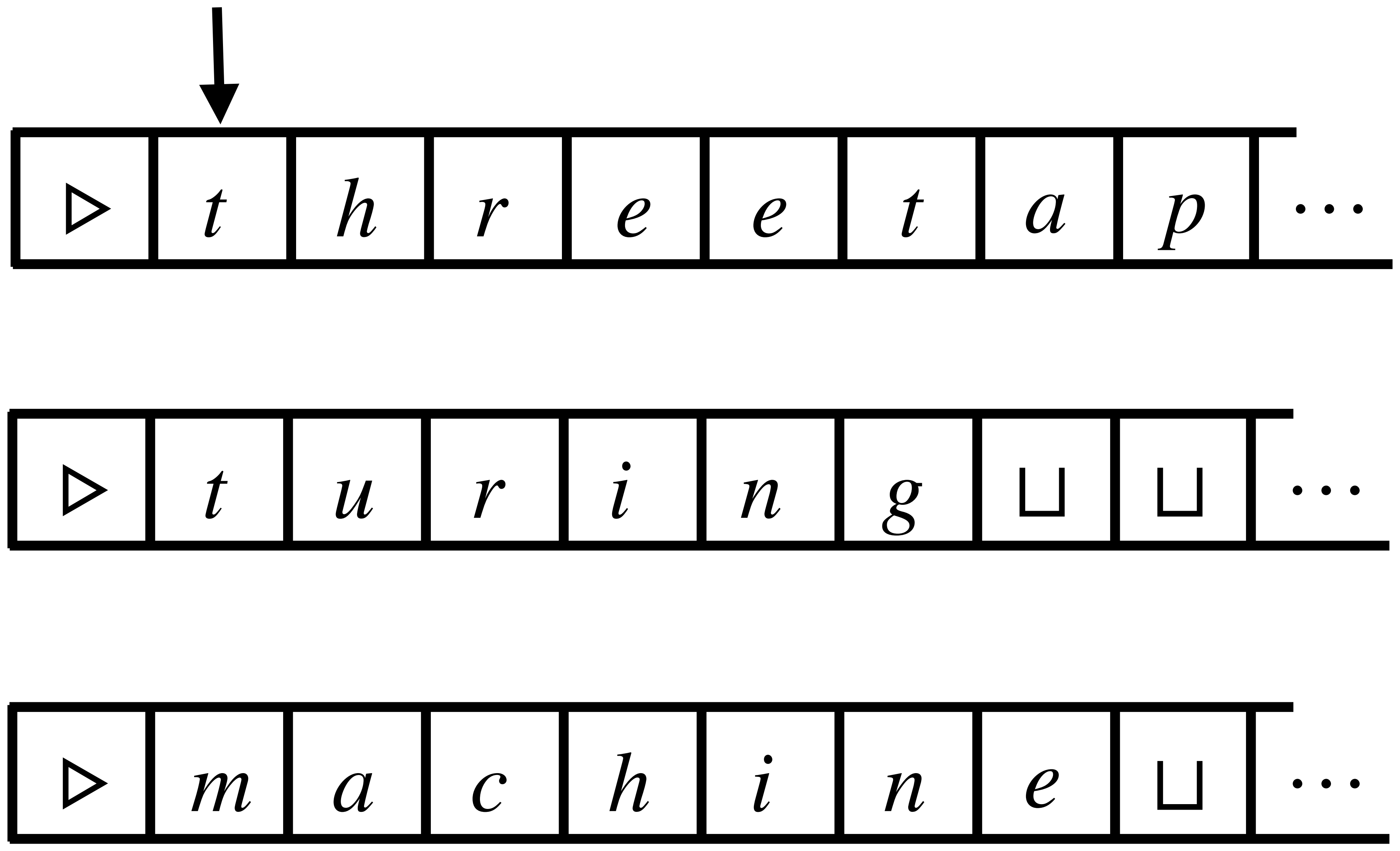

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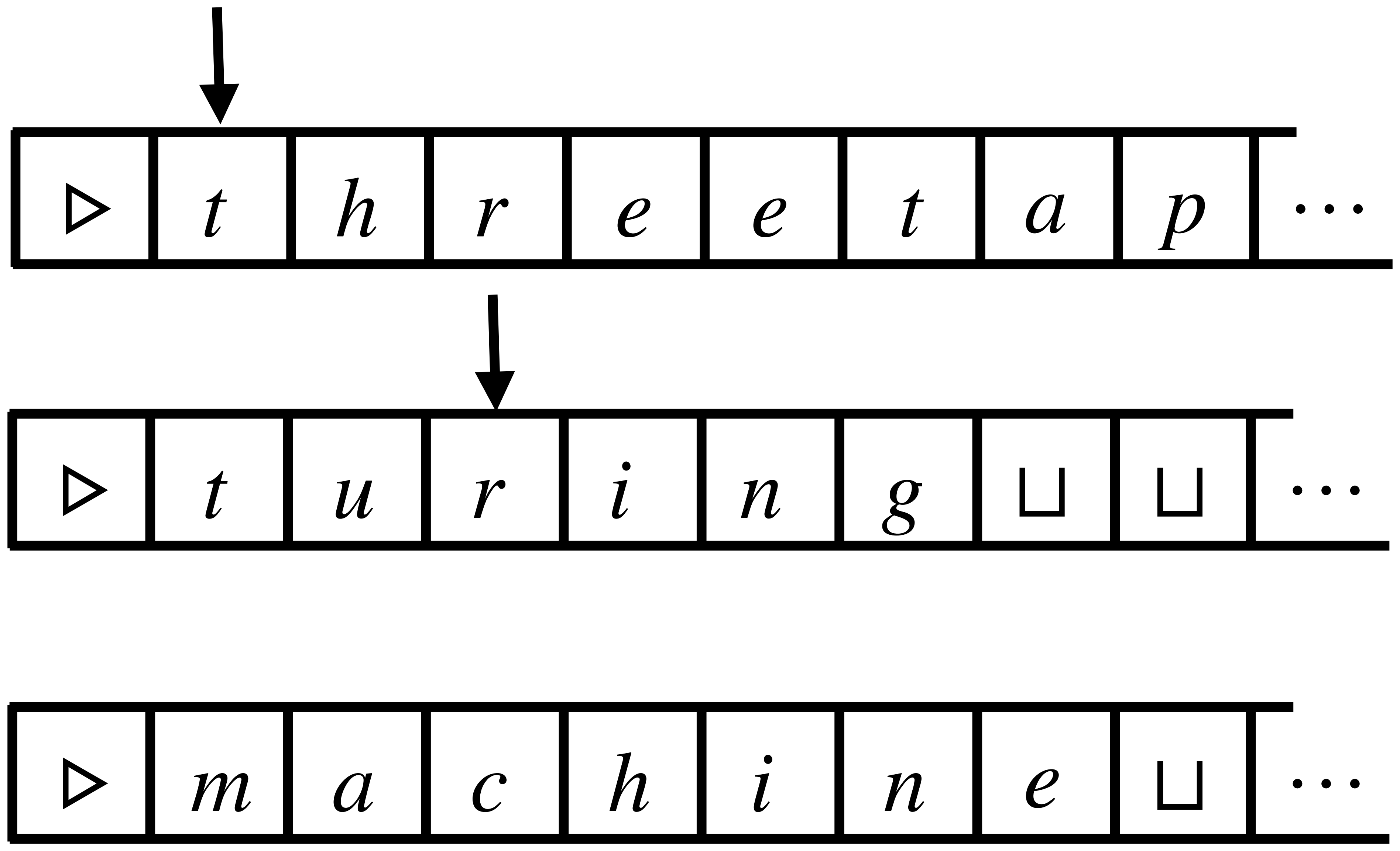

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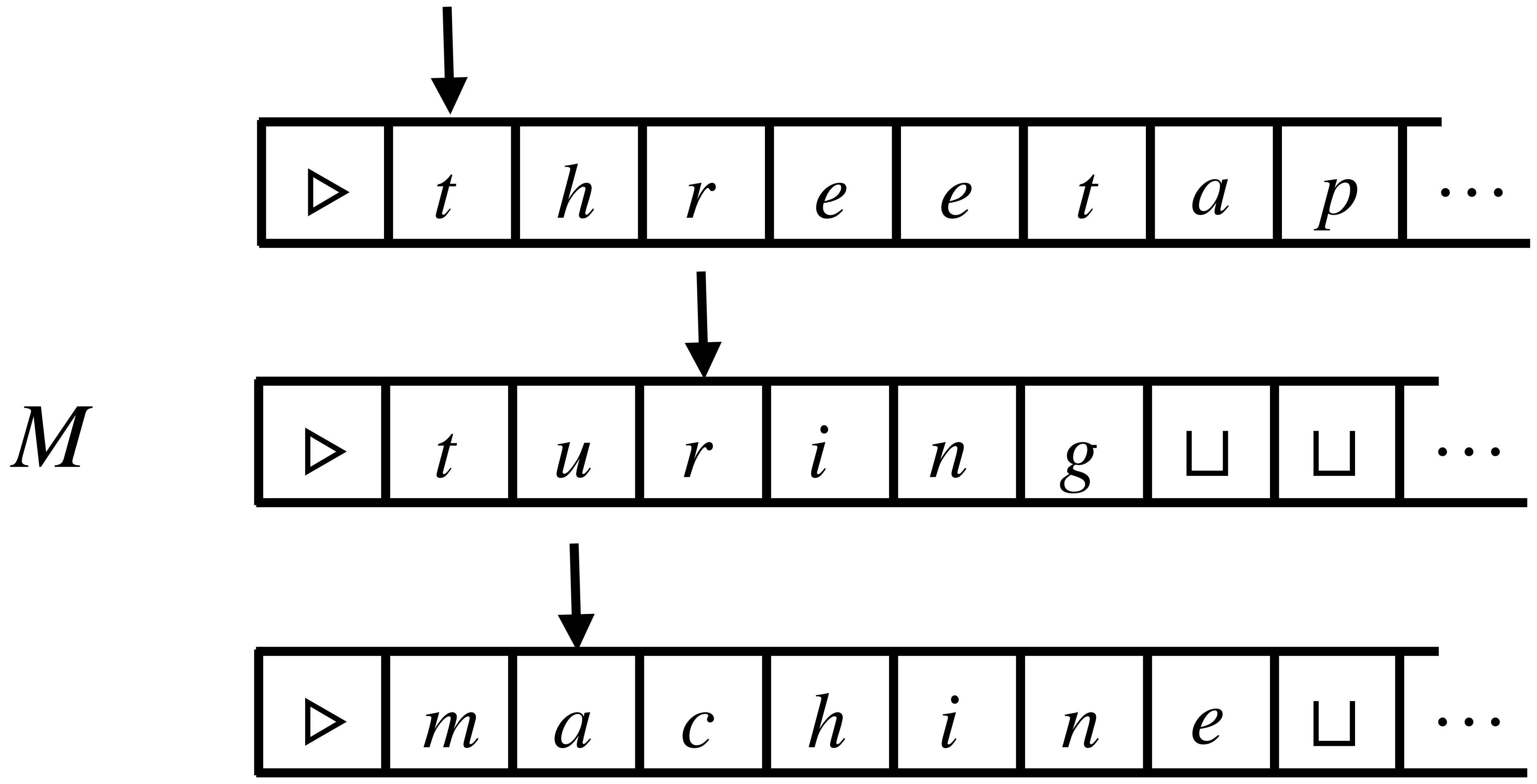

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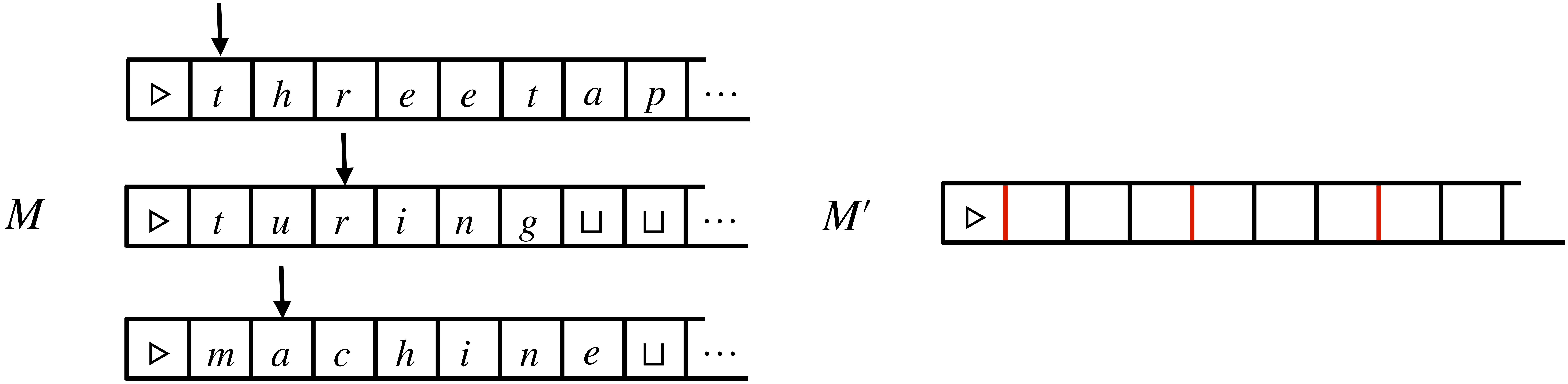

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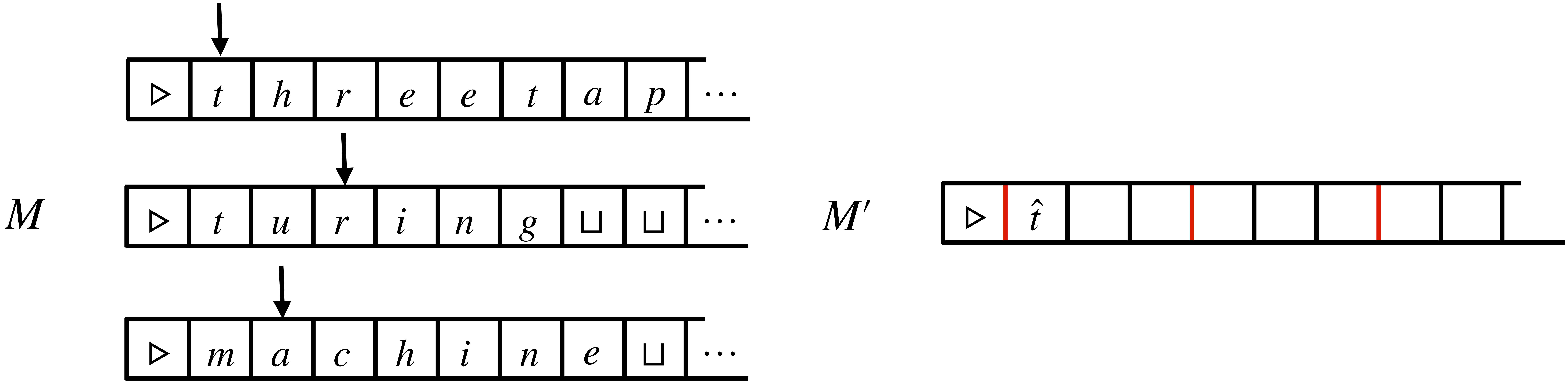

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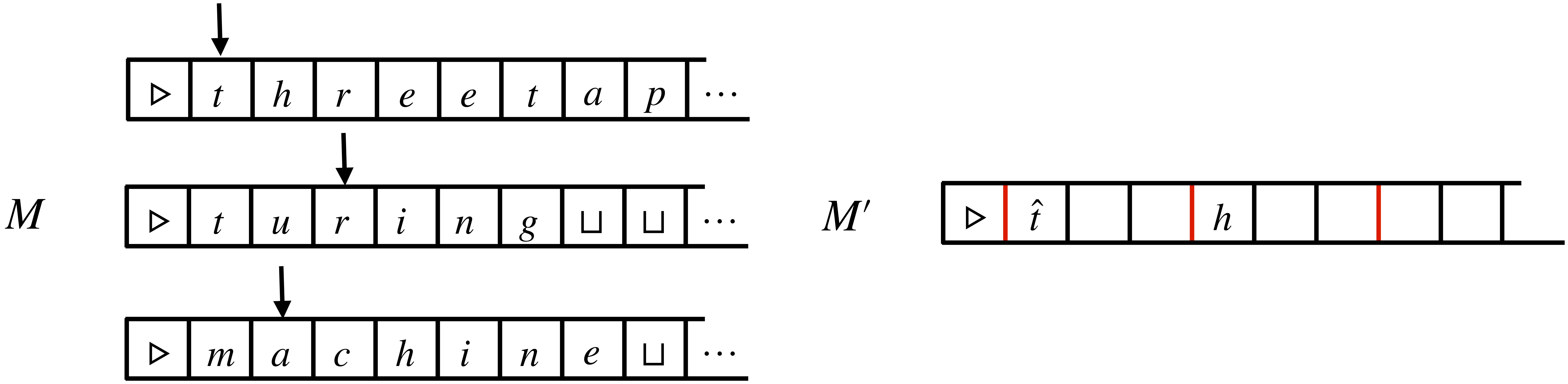

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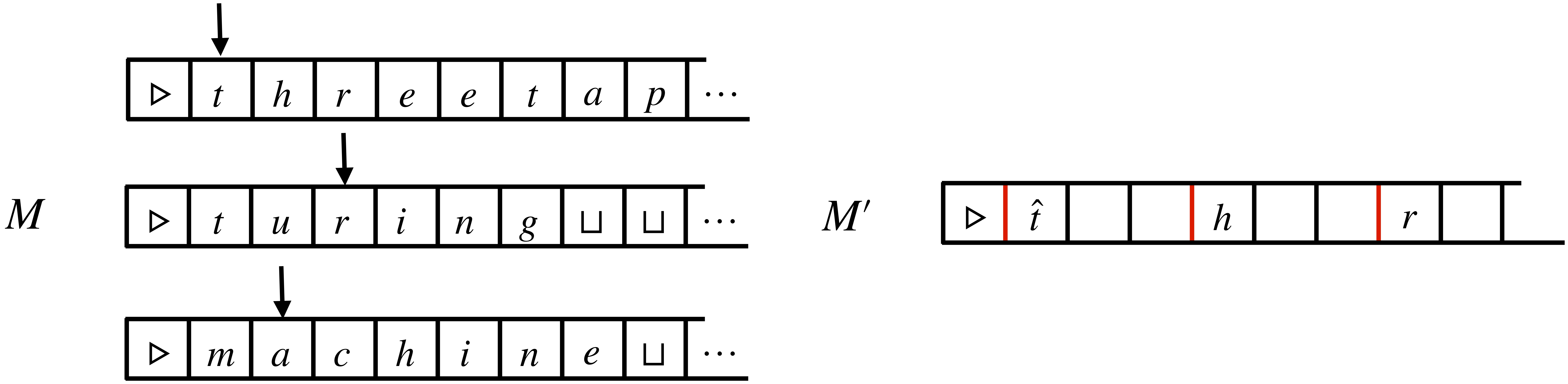

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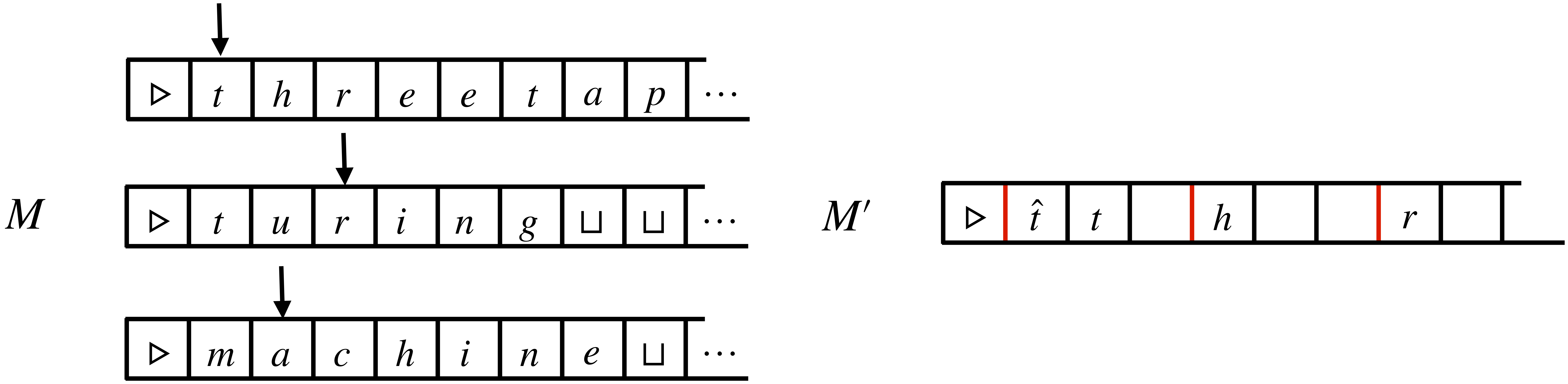

TMs with Single Tape

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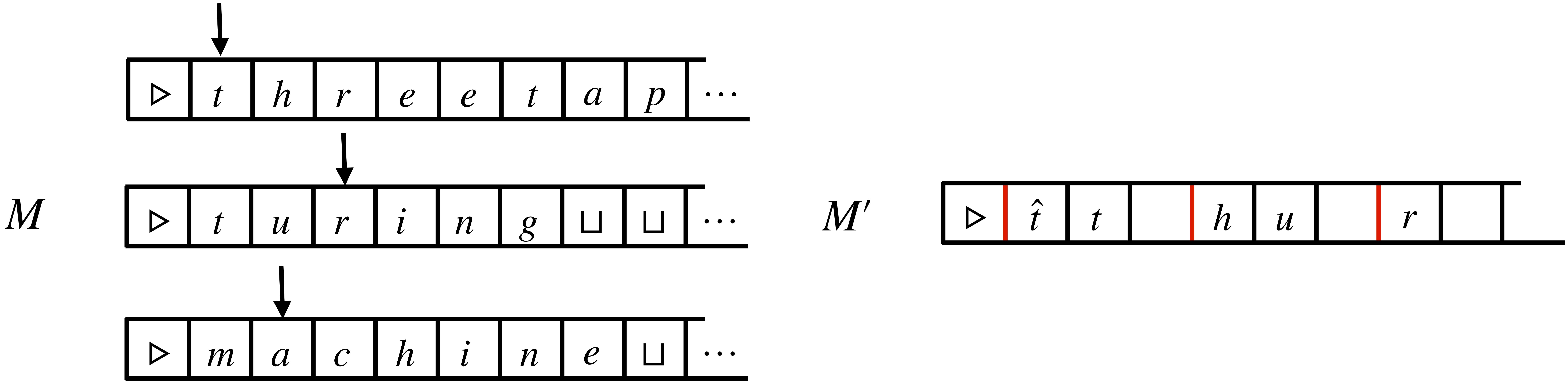

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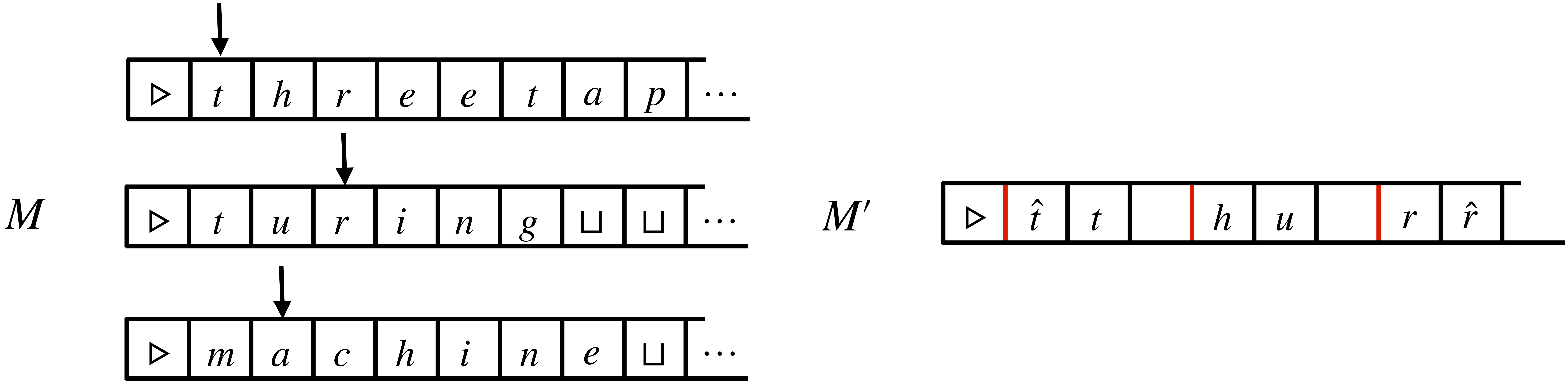

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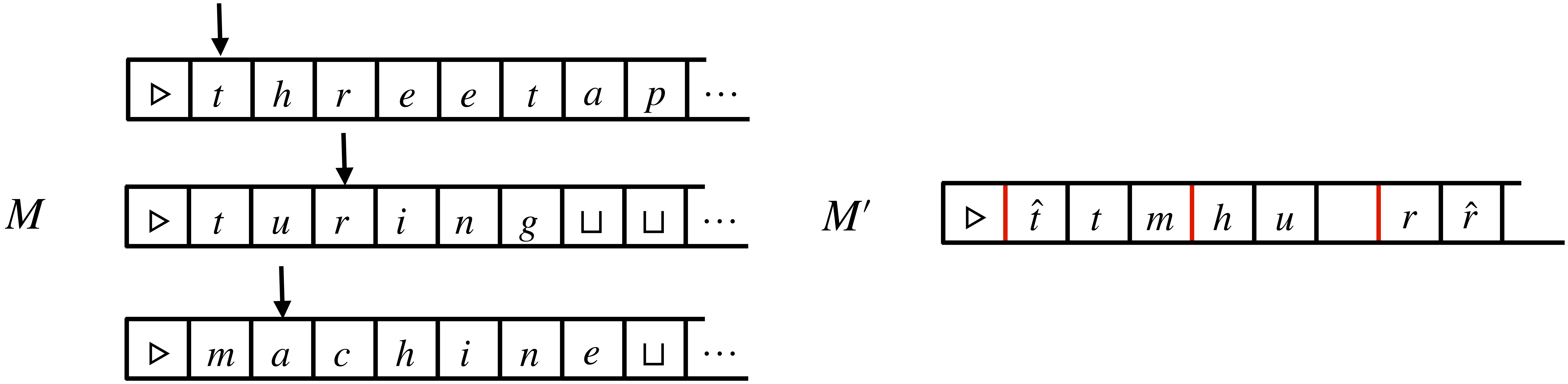

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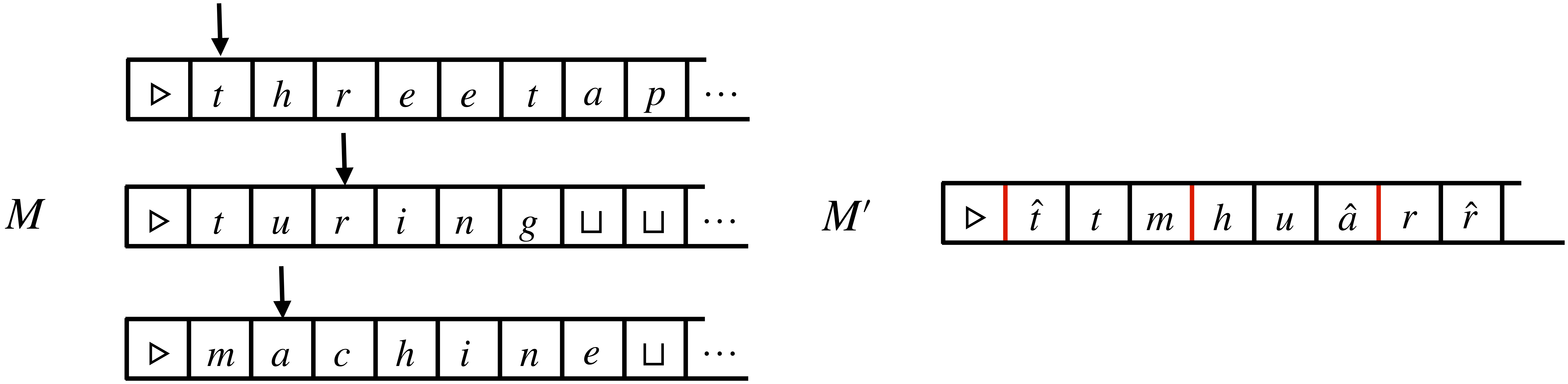

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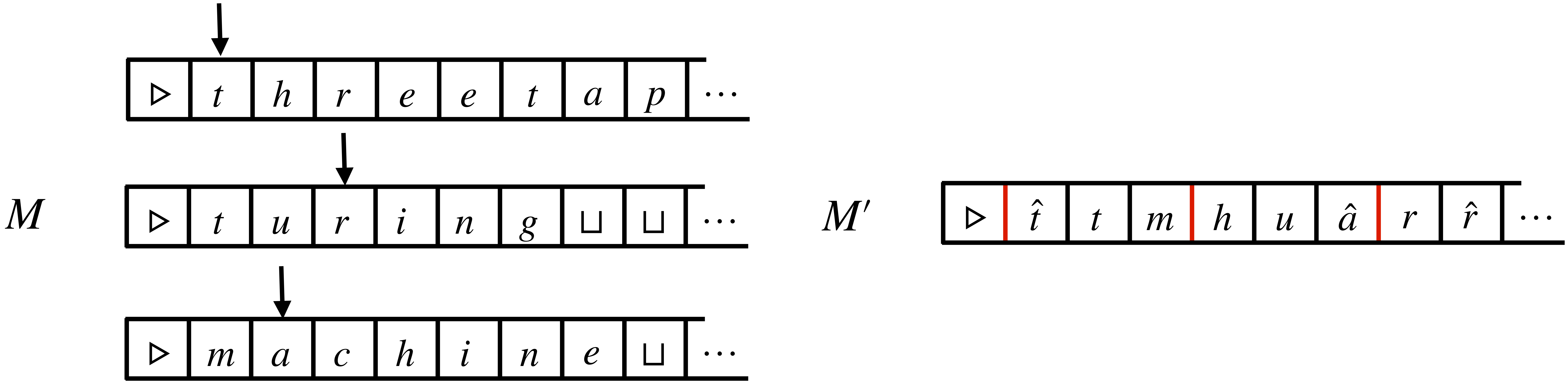

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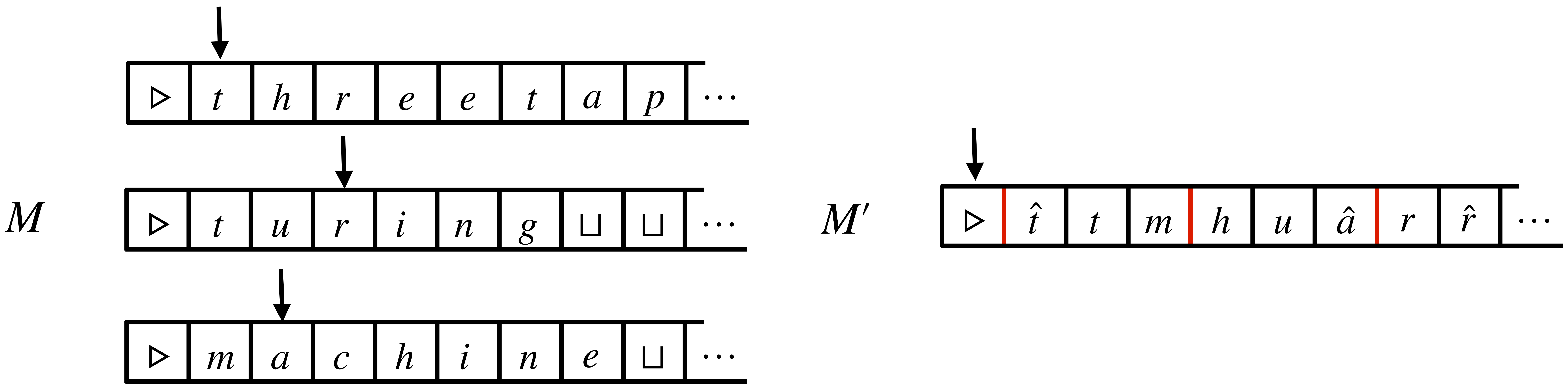

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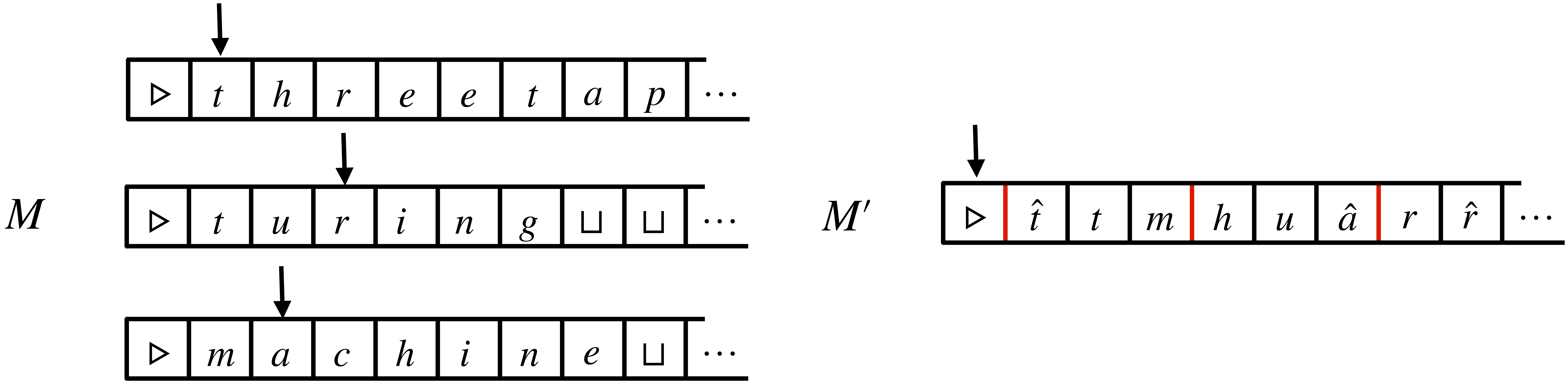

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- Sweep the tape in left to right direction and store k symbols in the current state.

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Proof: Simulating one step of M in M' :

- Sweep the tape in left to right direction and store k symbols in the current state.
- Use M's transition to determine the next state, symbols to write, and head movement and store this information in current state.

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Claim: For any $f : \{0,1\}^* \rightarrow \{0,1\}^*$ and time-constructible function $T : \mathbb{N} \rightarrow \mathbb{N}$, if f is computable in time $T(n)$ by a TM M using k tapes, then it is computable in time $O(k \,.\, T(n)^2)$ using a single-tape TM M'.

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- Sweep the tape in left to right direction and store k symbols in the current state.
- Use M's transition to determine the next state, symbols to write, and head movement and store this information in current state.
- Sweep the tape from right to left while updating the symbols.

TMs with Single Tape

Claim: For any $f : \{0,1\}^* \to \{0,1\}^*$ and time-constructible function $T : \mathbb{N} \to \mathbb{N}$, if f is computable in time $T(n)$ by a TM M using k tapes, then it is computable in time $O(k \, . \, T(n)^2)$ using a single-tape TM M'.

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